Sensitivity Analyses for Scenario Reduction in Flexible Flow Sheet Design with a Large Number of Uncertain Parameters

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A solution strategy for designing flexible process flow sheets with a large number of uncertain parameters is presented. The basic mathematical formulation is a two-stage stochastic program transformed into its multi-scenario deterministic equivalent. The main feature of the proposed approach is a tremendous reduction in scenarios to a smaller number of those critical ones. This reduction is achieved through simple sensitivity analyses that identify those uncertain parameters that are critical for feasibility and those that are involved in a stochastic approximation of the objective value. Through the application of this strategy, it is possible to solve the problems with several tens or even hundred uncertain parameters, assuming weak interactions between them. Feasible designs are obtained for a fixed degree of flexibility, while the expected objective function is approximated fairly well. The strategy is applied to two flow sheet examples.
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Introduction

Every optimal engineering design obtained by means of mathematical programming is only optimal for the specified values of input data used during the optimization. In reality, however, the majority of input parameters change frequently, e.g., model data (transfer, conversion and efficiency coefficients), process data (temperatures and pressures), and external data (demand and prices).[1] These fluctuations are responsible for changing optimal solutions into the suboptimal or even infeasible. Considering uncertainty in all engineering problems is thus of paramount importance.

The metrics of process flexibility was developed in the 1980s as a flexibility test and index.[2,3] Various approaches have been developed since then for optimization under uncertainty (for the review see Sahinids[4]).

An often used mathematical formulation is the two-stage stochastic programming with recourse, as introduced by Dantzig.[5] In this formulation, decision variables are partitioned into first-stage and second-stage variables. If the probability distributions of uncertain parameters are known, decision makers seek to determine the values of the first-stage variables with the best long term average performance criterion. The latter is usually expressed as the expected value of the economic criterion, e.g., the total cost, profit or the net present value. Evaluation of the expectancy requires multidimensional integration, which is usually performed through the scenario-based approach leading to a discretized deterministic equivalent problem. Acevedo and Pistikopoulos[6] described three integration schemes: Gauss–Legendre quadrature either with full scan of the uncertainty space or with preliminary evaluation of the feasible region, and Monte Carlo simulation.

Unfortunately, the number of scenarios increases rapidly with the number of uncertain parameters; therefore, a lot of effort has been made to solve such problems by means of approximations and simplifications. Raspanti et al.[7] used aggregation and smoothing functions to aggregate the constraints, simplify the models, and reduce the solution time. Some authors have proposed different sampling techniques, e.g., Wei and Realff [8] combined the sample average approximation (SAA) and outer approximation (OA) algorithms to solve stochastic Mixed Integer Nonlinear Programming (MINLP) problems, while Goyal and Ierapetritou[9] combined the simplicial-based approach with the SAA approach. Shastri and Diwekar[10] proposed an algorithm that integrates the traditional sampling method with the statistical reweighting technique, for speeding-up the optimization of large-scale nonlinear stochastic problems.

Several approximation techniques have also been developed, e.g., Novak Pintarič and Kravanja[11] used a predetermined subset of vertices to approximate the expected objective value of the reduced deterministic equivalent. Later, they proposed the Central Basic Point to approximate the expected value while feasibility is guaranteed by including the critical points.[12] Different strategies were also proposed for identifying the reduced set of critical points.[13] Karuppiah et al.[14] also solved the deterministic multi-scenario optimization problem within a reduced set of scenarios. The latter is determined through a special mixed-integer linear program (MILP), in which the main criterion is that the total probability of each uncertain parameter value

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over the reduced set of scenarios is equal, as primarily defined. Ostrovsky et al.[15] developed an approximate method for solving the two-stage optimization problem where the constraints must be satisfied with some probability (chance constraints).

Some authors avoided the scenario-based approach by applying a robust optimization which has the advantage that the size of the problem does not increase exponentially with the number of scenarios. Li and Ierapetritou[16] tested three robust counterpart optimization formulations for solving MINLP scheduling problems with uncertain prices, processing times, and demands. Assavapokee et al.[17] used a min–max regret robust optimization for those problems where uncertain information is represented by interval data, and illustrated the insufficiency of the robust solution obtained by using the endpoints of all compact intervals.

Despite significant progress being made toward solving engineering problems under uncertainty, the design and synthesis problems of large-scale chemical processes have not, as yet, been successfully solved. This problem is even more difficult in those biochemical processes which are subject to even higher uncertainty, while the reliability of models and input data is much lower.

The motivation of this work was to develop an approach for designing process flow sheets that contain a large number of uncertain parameters. This approach is based on the conventional two-stage stochastic formulation with recourse, and its transformation into a discretized deterministic equivalent problem (DEP). The main disadvantage of discretization by precise methods such as Gaussian quadrature or Monte Carlo simulation is that the set of discrete points becomes practically unmanageable for large numbers of uncertain parameters. The main goal of our work was thus to drastically reduce the set of discrete points (scenarios) in DEP that, on the one hand, would guarantee flexible optimal designs and, on the other hand, fair approximation of the stochastic expected objective value.

As techniques for scenario reduction usually involve rather complicated preparation procedures, the idea of this article is to use simpler preparation methods, such as sensitivity analyses, for this purpose. The analyses of sensitivity are typically undertaken during the initial uncertainty studies for determining the impacts of varying input data on the quality of the solution. The intention is to exploit the results of sensitivity analyses as much as possible for reducing the number of scenarios in the DEP, as shown briefly by Kasaš et al.[18], in order to be capable of solving the problems with up to a hundred uncertain parameters. We have restricted the proposed approach at this early stage of the development to only those flow sheet problems where none or weak interactions between uncertain parameters were assumed. Statistical methods for measuring the levels of interactions between such large numbers of parameters are computationally very expensive. Measuring the interactions in this work was, therefore, limited to small problems.

It should be mentioned that similarly, as the interactions cannot be measured for problems with large numbers of uncertain parameters, also the flexibility cannot be tested by known methods for the flexibility test and index. As the assumption of no or weak interactions cannot be held true in general, the flexibility of solutions cannot be guaranteed in the exact way; however, the flexibility and optimality of the design obtained can still be significantly improved by using the proposed approach. Two flow sheet examples are presented to illustrate these challenges: (a) design of a flexible heat exchanger network (HEN) for which the executions of the exact stochastic approach and the flexibility index were possible and (b) design of a flexible bioethanol plant for which the executions of rigorous approaches and flexibility measures were impossible, so the approximate approach was one of the rare ways to generate (near) optimal flexible design.

Formulation of Significantly Reduced Deterministic Equivalent Problem

General flow sheet design problem under uncertainty

Assume having a general flow sheet design problem modeled as a nonlinear programming problem (NLP)

$$\begin{aligned} Z=&\max_{\mathbf{d},\mathbf{x},\mathbf{z}} \left[-f^0(\mathbf{d})+f(\mathbf{x},\mathbf{z},\boldsymbol{\theta})\right] \\ \text{s.t.}\quad & \mathbf{h}(\mathbf{d},\mathbf{x},\mathbf{z},\boldsymbol{\theta})=0 \\ & \mathbf{g}(\mathbf{d},\mathbf{x},\mathbf{z},\boldsymbol{\theta})\leq 0 \\ & \mathbf{d}\geq \mathbf{g}_{\mathrm{d}}(\mathbf{x},\mathbf{z},\boldsymbol{\theta}) \\ & \mathbf{d},\mathbf{x},\mathbf{z}\in\Re^+, \qquad \boldsymbol{\theta}^{\mathrm{LO}}\leq\boldsymbol{\theta}\leq\boldsymbol{\theta}^{\mathrm{UP}} \end{aligned} \tag{NLP}$$

where **x**, **z**, **d** represent the vectors of continuous state, control, and design variables, and **h**, **g**, $\mathbf{g}_{\mathrm{d}}$ are the vectors of equality, inequality, and design specification constraints. Z is the objective variable in which f^0 represents a term dependent on the design variables only, e.g., fixed and variable investment costs, while f depends on the operating and control variables, e.g., the operating cost. The components of the uncertain parameters' vector $\boldsymbol{\theta}$, can vary between predefined lower and upper bounds, $\boldsymbol{\theta}^{\mathrm{LO}}$ and $\boldsymbol{\theta}^{\mathrm{UP}}$. As uncertain parameters can occupy any value between their bounds, the (NLP) problem corresponds to an infinite optimization problem which cannot be solved directly. One alternative way is to transform the infinite (NLP) problem into its deterministic equivalent problem by using a finite set of discretized values for uncertain parameters.

General formulation of a deterministic equivalent problem

In the (NLP) formulation, the variables can be divided into two groups: the first-stage variables that have to be determined before any information about uncertainties becomes available, and the second-stage variables that can be determined after the uncertainty is resolved. In the case of process flow sheet design, the typical first-stage variables are the design variables (sizes of process units), while the second-stage variables are associated with operating decisions, such as flow rates, pressures, and temperatures. Such a formulation is known as a two-stage stochastic formulation with recourse, and is solved through transformation into its deterministic equivalent problem (DEP), by discretizing the uncertain parameters with a finite set of scenarios. A fixed degree of flexibility is assumed between predefined lower and upper bounds of the uncertain parameters. Let S be the set of selected discrete points, $S=\{s,\ s=1,\ \ldots,\ N_{\mathrm{DP}}\}$. Then the general DEP for maximizing a net present value is as follows

$$\begin{aligned} \mathrm{EZ}=&\max_{\mathbf{d},\mathbf{x}_s,\mathbf{z}_s}\left[-f^0(\mathbf{d})+\sum_{s=1}^{N_{\mathrm{DP}}}\mathbf{p}_s f(\mathbf{x}_s,\mathbf{z}_s,\boldsymbol{\theta}_s)\right] \\ \text{s.t.}\quad & \left.\begin{aligned} &\mathbf{h}(\mathbf{d},\mathbf{x}_s,\mathbf{z}_s,\boldsymbol{\theta}_s)=0 \\ &\mathbf{g}(\mathbf{d},\mathbf{x}_s,\mathbf{z}_s,\boldsymbol{\theta}_s)\leq 0 \\ &\mathbf{d}\geq\mathbf{g}_{\mathrm{d}}(\mathbf{x}_s,\mathbf{z}_s,\boldsymbol{\theta}_s) \end{aligned}\right\}\quad s\in S \\ & \mathbf{d},\mathbf{x}_s,\mathbf{z}_s\in\Re^+ \end{aligned} \tag{DEP}$$

In the (DEP), the expected value of the objective variable, EZ, is maximized, using those discrete probability values, $\mathbf{p}_s$, associated with each scenario s of uncertain parameters, $\boldsymbol{\theta}_s$. The crucial issue in solving (DEP) is how to select a set of discrete scenarios S. Precise approaches such as the Gaussian quadrature or the Monte Carlo simulation use a huge number of scenarios—the zeros of the Legendre polynomials (Gaussian quadrature) or randomly selected points (Monte Carlo). Consequently, the results obtained are more accurate but at the cost of significantly higher computational effort, because the number of discrete points increases drastically with uncertain parameters. A reduction in scenarios is, therefore, essential for solving large scale process flow sheets with great numbers of uncertain parameters. A heuristic methodology for curbing tremendous scenario growth is presented in the following subsections.

Reduced deterministic equivalent for approximate stochastic optimization

In the method employed in this work, the reduced deterministic equivalent is solved over the set of scenarios consisting of a minimal set of critical points for ensuring flexibility, and one single point (the Central Basic Point) for approximating the expected value.[12] Assuming that the Central Basic Point (CBP or $\boldsymbol{\theta}^{\mathrm{CBP}}$) and the set of critical points C can be determined in advance, a significantly reduced nonlinear deterministic equivalent problem (RDEP) could be solved over a union of the scenarios $C \cup \boldsymbol{\theta}^{\mathrm{CBP}}$

$$\begin{aligned} &\mathrm{EZ} \approx \max_{\mathbf{d},\mathbf{x}_s,\mathbf{z}_s} \left[-f^0(\mathbf{d})+f(\mathbf{x}_s,\mathbf{z}_s,\boldsymbol{\theta}_s)\right]_{s=\mathrm{CBP}} \\ &\text{s.t.} \quad \left.\begin{aligned} &\mathbf{h}(\mathbf{d},\mathbf{x}_s,\mathbf{z}_s,\boldsymbol{\theta}_s)=0 \\ &\mathbf{g}(\mathbf{d},\mathbf{x}_s,\mathbf{z}_s,\boldsymbol{\theta}_s)\le 0 \\ &\mathbf{d} \ge \mathbf{g}_{\mathrm{d}}(\mathbf{x}_s,\mathbf{z}_s,\boldsymbol{\theta}_s) \end{aligned}\right\} \quad s \in C \cup \boldsymbol{\theta}^{\mathrm{CBP}} \qquad \text{(RDEP)} \\ &\qquad\ \mathbf{d},\mathbf{x}_s,\mathbf{z}_s \in \Re^+ \end{aligned}$$

The expected objective variable EZ in (RDEP) is approximated at the single scenario θ^{CBP}, thus avoiding the rigorous Gaussian integration. The objective function and the constraints are coupled through the first-stage design variables $\mathbf{d}$, which must guarantee a flexibility of optimal design for the entire uncertainty space. This goal is achieved by using a minimum set of scenarios, called critical points $s \in C$, for which some of the constraints $\mathbf{g}$ or $\mathbf{g}_{\mathrm{d}}$ will be active. If the constraints are convex, then the critical points lie at the limiting values of uncertain parameters' intervals. Halemane and Grossmann[2] showed that in convex problems, the design that is feasible for its critical points will also be feasible for all the other values of uncertain parameters within their intervals. There can also be nonconvex problems for which critical points are sufficient to meet required flexibility, however, this cannot generally be true, and designs obtained from such models need to be tested for flexibility.

Preparation procedures should be performed before solving the above problem, in order to identify the critical points, and determine the Central Basic Point. Both steps are presented in the next two sections. As the intention of this work was to avoid complicated procedures for scenario reduction, it is shown that these two goals can be achieved by analyzing the system's sensitivity to the variations of various input data.

Identification of critical points

Critical points are defined as those worst-case scenarios that ensure feasible design over the whole uncertainty region. A sensitivity analysis of uncertain parameters' influences on design variables is performed in order to identify these points. It was assumed in this article that probability distributions of uncertain parameters are either independent (uncorrelated) or perfectly correlated. Perfectly correlated parameters can be grouped and handled as one uncertain parameter, e.g., the heat-transfer coefficients of all heat exchangers or the tray efficiencies of all the separation columns. It was also assumed that the interactions between parameters are weak or nonexistent.

Sensitivity Analyses. A sensitivity analysis is performed by varying each uncertain parameter for a selected step-size between its lower and upper bounds, while other parameters are fixed at the nominal values, i.e., the one-at-a-time sensitivity test. Let I be the set of all uncertain parameters, $I=\{i, i=1,\ldots, N_{\mathrm{UP}}\}$, and J the set of selected points between the lower and upper bounds, $J=\{j, j=1,\ldots,N_{\mathrm{SP}}\}$. Then the problem (P1) is solved sequentially as one-scenario problem for $N_{\mathrm{UP}} \cdot N_{\mathrm{SP}}$ times

$$\begin{aligned} &Z_{i,j}=\max_{\mathbf{d}_{ij},\mathbf{x}_{ij},\mathbf{z}_{ij}} \left[-f^0\left(\mathbf{d}_{i,j}\right)+f\left(\mathbf{x}_{i,j},\mathbf{z}_{i,j},\theta_{i,j},\boldsymbol{\theta}^{\mathrm{N}}_{I\setminus i}\right)\right] \\ &\text{s.t.} \quad \mathbf{h}\left(\mathbf{d}_{i,j},\mathbf{x}_{i,j},\mathbf{z}_{i,j},\theta_{i,j},\boldsymbol{\theta}^{\mathrm{N}}_{I\setminus i}\right)=0 \\ &\qquad \mathbf{g}\left(\mathbf{d}_{i,j},\mathbf{x}_{i,j},\mathbf{z}_{i,j},\theta_{i,j},\boldsymbol{\theta}^{\mathrm{N}}_{I\setminus i}\right)\le 0 \\ &\qquad \mathbf{d}_{i,j}=\mathbf{g}_{\mathrm{d}}\left(\mathbf{x}_{i,j},\mathbf{z}_{i,j},\theta_{i,j},\boldsymbol{\theta}^{\mathrm{N}}_{I\setminus i}\right) \qquad \text{(P1)}_{i,j} \\ &\qquad \mathbf{d}_{i,j},\mathbf{x}_{i,j},\mathbf{z}_{i,j} \in \Re^+ \qquad \forall i \in I, \forall J \in J \end{aligned}$$

In a single-scenario problem (P1), the value of ith uncertain parameter under consideration is fixed at the selected value $\theta_{i,j}$, while the remaining parameters are fixed at the nominal values $\boldsymbol{\theta}^{\mathrm{N}}_{I\setminus i}$. The parameter under consideration sequentially obtains j different values/scenarios between its lower and upper bounds, and (P1) is solved for each scenario $\theta_{i,j}$, $j=1,\ldots, N_{\mathrm{SP}}$. After solving (P1) for several values of a particular parameter, the next uncertain parameter is analyzed.

The number of variating points N_{SP} should be selected so that the parameter's space is scanned sufficiently, and that the computational effort is kept at a reasonable level. In convex problems, critical points correspond to the extreme points of uncertain parameters, and three values (e.g., lower, nominal, and upper) of each parameter would suffice for establishing its influence on the design and objective variables. In nonconvex problems, where critical points could lie also at the nonvertex points, we suggest using 5 to 10 equidistant values in order to detect potential nonvertex critical points. Note that (P1) represents a one-dimensional (1-D) (one scenario) problem, so its solving could be repeated iteratively in a loop over the space of each uncertain parameter without huge computational effort. For example, if the problem would contain 100 uncertain parameters, and 10 values would be chosen for each, then (P1) should be solved in a loop for thousand times.

Based on the results of the sensitivity analysis, the effects of changing parameters on design variables are studied next.

The Effects of Uncertainty on Design Variables. Assume K is a set of design variables, $K=\{k, k=1,\ldots, N_{\mathrm{DV}}\}$. The influences of varying uncertain parameters on design variables can be classified into three groups: none, monotonic (positive or negative), and nonmonotonic:

a. Uncertain parameters, θ_i, with no influence on the design variables, are those for which sensitivity analysis revealed

$$\frac{d_{k,i,j+1}-d_{k,i,j}}{\theta_{i,j+1}-\theta_{i,j}}=0 \quad \forall k\in K, \quad i\in I, \quad \forall j=1,\ldots,N_{\mathrm{SP}}-1 \quad (1)$$

The strict equality to zero could be relaxed for practical use so that the left-hand side is less or equal to some small positive constant. Uncertain parameters with no influence on the design variables are usually various economic data, such as prices, interest and tax rates, lifetime and depreciation periods, etc. These parameters can be fixed to any value in the critical points; however, the nominal values would be used in most cases.

b. Uncertain parameter θ_i has a positive monotonic influence on the design variable d_k, if the following is true

$$\frac{d_{k,i,j+1}-d_{k,i,j}}{\theta_{i,j+1}-\theta_{i,j}}\geq 0 \quad \forall k\in K, \quad i\in I, \quad \forall j=1,\ldots,N_{\mathrm{SP}}-1 \quad (2)$$

The uncertain parameter with positive monotonic influence on some design variables and no influence on the others, can be fixed to its upper bound, θ_i^{UP}, in the critical points during (RDEP) optimization.

Negative monotonic influence is recognized in a similar way

$$\frac{d_{k,i,j+1}-d_{k,i,j}}{\theta_{i,j+1}-\theta_{i,j}}\leq 0 \quad \forall k\in K, \quad i\in I, \quad \forall j=1,\ldots,N_{\mathrm{SP}}-1 \quad (3)$$

If Eq. 3 holds for all design variables, then uncertain parameter can be fixed to its lower bound, θ_i^{LO}. Monotonic parameters are often product demand, heat-transfer coefficients, efficiencies, etc.

c. Uncertain parameter θ_i has a nonmonotonic influence on design variables if any of the Eqs. 2 or 3 hold for different j or k

$$\left.\begin{array}{l}\dfrac{d_{k,i,j+1}-d_{k,i,j}}{\theta_{i,j+1}-\theta_{i,j}}\geq 0\\ \dfrac{d_{k,i,j+1}-d_{k,i,j}}{\theta_{i,j+1}-\theta_{i,j}}\leq 0\end{array}\right\} k\in K, \quad i\in I, \quad j=1,\ldots,N_{\mathrm{SP}}-1 \quad (4)$$

In practice, this means that some design variables increase or decrease monotonically, the others changes nonmonotonically, and the thirds remain practically unchanged with varying values of particular uncertain parameter. Those parameters with nonmonotonic influence participate in a procedure for determining the minimum set of critical points, as described in the next section.

Merging the Parameters with Nonmonotonic Influence into a Minimum Set of Critical Points. Suppose that the results of sensitivity analysis identify the reduced subset of uncertain parameters, $I' \subset I$, that affect the design variables in nonmonotonic way. The following matrix is formed for such uncertain parameters

$$A=\left[a_{k,i}\right] \quad k\in K, i\in I' \quad (5)$$

where

$$a_{k,i}=\begin{cases}0, & \text{if } \theta_i \text{ has no influence on } d_k\\ 1, & \text{if } d_k^{\max} \text{ at } \theta_i^{\mathrm{LO}}\\ 2, & \text{if } d_k^{\max} \text{ at } \theta_i^{\mathrm{UP}}\\ 3, & \text{if } d_k^{\max} \text{ at } \theta_i \text{ and } \theta_i^{\mathrm{LO}}<\theta_i<\theta_i^{\mathrm{UP}}\end{cases} \quad (6)$$

The value 1 in the matrix A expresses a negative monotonic influence of uncertain parameter i on the design variable k, the value 2 the positive monotonic influence, and the value 3 the nonmonotonic influence. The values in the matrix A are merged into the minimum set of points so that all the combinations of uncertain parameters are covered. In order to achieve this goal, the procedure was developed based on the "Algorithm for merging the active bounds in the minimum number of points."[19] This procedure was extensively described in the literature[13], while the main steps can be summarized as follows:

a. If there exists a column in A in which only values 0 and 1, or 0 and 2, or 0 and 3 appear, then the entire column should be fixed at the values 1, 2, or 3, respectively. Note that this rule coincides with the Eqs. 2 and 3.

b. If there exists a pair of rows in A which are identical in some elements, but the unequal elements in one row are all zeros, then this row could be eliminated from the matrix A.

c. The rows with all nonzero elements represent critical points.

d. All possible combinations should be created from the remaining rows (with some zero elements) by substituting the zeros with the values 1, 2, and 3. A minimum set of different combinations should be found that cover all rows with some zero elements, and the rows with all nonzero elements. This task is trivial for small examples, however, in the larger problems it could be accomplished by solving the integer programming set-covering problem.

e. The values 1, 2, and 3 in the remaining rows should be finally transformed to the actual values of uncertain parameters, considering that 1 represents the lower bound, 2 the upper bound, and 3 the intermediate value, as shown in Eq. 6.

The result of this step is a rather smaller set of critical points, $C=\{c, c=1,\ldots, N_{\mathrm{CP}}\}$, that ensure flexibility of the design obtained by solving (RDEP). In practice, this means that each critical point covers as many first-stage design variables (equipment sizes) in a process flow sheet as possible.

Limitations of the proposed approach. Determining critical points through sensitivity analyses relies on changing the values of each uncertain parameter, i.e., one at a time, and fixing other uncertain parameters at the nominal values. This is a significant simplification that is based on the assumption that the interactions between uncertain parameters are not present. Noninteracting parameters affect the design variables independently of the other parameters, and one-at-a-time sensitivity analysis would be sufficient. This applies to the models with linear or almost fully additive nonlinear structures. If the parameters interact significantly in the nonlinear ways, the results obtained and the trends established could be different when fixing uncertain parameters at other values than the nominal. This could be especially problematic if the nominal value of the parameter were zero, thus eliminating its interactions entirely. In process flow sheet design, however, the uncertain parameters would typically have positive (nonzero) nominal values, so it is assumed in this work that the interactions between the parameters would not significantly affect those trends established during the sensitivity analysis.

An upgrade of the proposed methodology for significantly interacting uncertain parameters is under way. The interactions could be quantified by well-known variance-based methods, e.g., by a comparison between the total effect and the main effect.[20] Main effect (or the first order sensitivity index) measures the importance of each parameter alone, while the total effect measures the effect of parameter and its interactions. If the sum of the main effects is equal (or close) to one, no (or weak) interactions between uncertain parameters are present. The most interacting parameters are those with large total effects, and large differences between the total and main effects.

These methods are computationally extremely intensive, and alternative approaches need to be developed for very large problems. Our recent experiences have shown that even the minimum influence of an uncertain parameter on a design variable should not be ignored in examples with significant nonlinear interactions between uncertain parameters. In such cases, even small nonzero gradients in the sensitivity analysis should be respected in order to identify the correct critical points, meaning that the criterion of nonexistent interactions, defined by Eq. 1, should be strictly set to zero.

Determination of the central basic point

After determining those critical points that ensure feasible operation and flexible design, we can proceed to the approximation of the expected objective function. The expected value of the objective function can be approximated at the nominal point if all uncertain parameters are described by symmetrical distribution functions, and have either none or linear influence on the objective variable. Otherwise, the deviation from the nominal point is considered for those parameters with nonlinear influences, by determining the Central Basic Point (CBP).

The Effects of Uncertainty on the Objective Variable. An analysis of uncertain parameters' influences on the objective variable is performed in order to identify those parameters that cause deviations in the objective value from its nominal value. The results of the sensitivity analyses are used to accomplish this task. The influences of varying uncertain parameter θ_i on the objective variable can be classified into three groups: none, linear, and nonlinear:

a. Uncertain parameters θ_i with no influence on the objective variable are those for which sensitivity analysis reveals

$$\frac{Z_{i,j+1}-Z_{i,j}}{\theta_{i,j+1}-\theta_{i,j}}=0 \quad i\in I, \quad \forall j=1,\dots,N_{\text{SP}}-1 \tag{7}$$

Again, the strict equality could be relaxed into inequality by introducing a small positive constant. Those uncertain parameters with no influence on objective variable can be fixed to their nominal values for approximating the expected objective function during (RDEP) optimization.

b. The linear influence of the uncertain parameter θ_i on the objective variable is recognized if equal successive gradients are identified

$$\frac{Z_{i,j}-Z_{i,j-1}}{\theta_{i,j}-\theta_{i,j-1}}=\frac{Z_{i,j+1}-Z_{i,j}}{\theta_{i,j+1}-\theta_{i,j}} \quad i\in I, \quad \forall j=2,\dots,N_{\text{SP}}-1 \tag{8}$$

Those uncertain parameters with linear influences on the objective variables can be fixed to their nominal values during RDEP optimization, as it could be assumed that those variations in negative directions would be balanced by those in positive directions. It should be noted that fixing a parameter to its nominal value is only correct for those parameters with symmetrical probability distributions, while in the case of asymmetrically distributed parameters this would introduce some error into the result. The size of the error is under investigation.

c. The nonlinear influence of uncertain parameter θ_i is recognized when

$$\frac{Z_{i,j}-Z_{i,j-1}}{\theta_{i,j}-\theta_{i,j-1}}\neq\frac{Z_{i,j+1}-Z_{i,j}}{\theta_{i,j+1}-\theta_{i,j}} \quad i\in I, \quad j=2,\dots,N_{\text{SP}}-1 \tag{9}$$

Those parameters with nonlinear influences participate in a procedure for calculating the Central Basic Point, while others are fixed to the nominal values within the CBP.

Calculation of the Central Basic Point. This procedure relies on the calculations of conditional expectations for those uncertain parameters with nonlinear influence on the objective variable. The conditional expectations are calculated through the decomposition of a multi-dimensional uncertain problem into several 1-D ones. The procedure was described by Novak Pintarič and Kravanja[12], and is only briefly summarized here.

Suppose G is a set of Gaussian quadrature points, $G=\{g, g=1, \dots, N_{\text{GP}}\}$, and $I'' \subset I$ is a reduced subset of those uncertain parameters with nonlinear influences on the objective function. Then the problem $(\text{P2})_i$ is solved for each nonlinear uncertain parameter θ_i at a set of scenarios composed of critical points and Gaussian quadrature points of this particular parameter, while other parameters are fixed at the nominal values $(\boldsymbol{\theta}^{\text{N}}_{I\backslash i})$

$$\begin{aligned}
\text{E}Z_i=&\max_{\mathbf{d}_i,\mathbf{x}_{i,s},\mathbf{z}_{i,s}}\left[-f^0(\mathbf{d}_i)+\sum_{s\in G}\mathbf{p}_s f\left(\mathbf{x}_{i,s},\mathbf{z}_{i,s},\theta_{i,s},\boldsymbol{\theta}^{\text{N}}_{I\backslash i}\right)\right]\\
\text{s.t.}\quad & \left.\begin{aligned}
&\mathbf{h}\left(\mathbf{d}_i,\mathbf{x}_{i,s},\mathbf{z}_{i,s},\theta_{i,s},\boldsymbol{\theta}^{\text{N}}_{I\backslash i}\right)=0\\
&\mathbf{g}\left(\mathbf{d}_i,\mathbf{x}_{i,s},\mathbf{z}_{i,s},\theta_{i,s},\boldsymbol{\theta}^{\text{N}}_{I\backslash i}\right)\leq 0\\
&\mathbf{d}_i\geq \mathbf{g}_{\text{d}}\left(\mathbf{x}_{i,s},\mathbf{z}_{i,s},\theta_{i,s},\boldsymbol{\theta}^{\text{N}}_{I\backslash i}\right)
\end{aligned}\right\} \quad s\in C\cup G, i\in I''\\
& \mathbf{d}_i,\mathbf{x}_{i,s},\mathbf{z}_{i,s}\in\Re^{+}
\end{aligned} \tag{(P2i)}$$

The results of this step are the conditional expected values of the nonlinear parameters, $\text{E}Z_i$. In addition, a series of objective values at all Gaussian quadrature points can be calculated

$$Z_{i,s}=-f^0(\mathbf{d}_i)+f\left(\mathbf{x}_{i,s},\mathbf{z}_{i,s},\theta_{i,s},\boldsymbol{\theta}^{\text{N}}_{I\backslash i}\right) \qquad s\in G, \quad i\in I'' \tag{10}$$

Based on these results, a regression function, r_i, is derived at, that has the best fit for the series of points, $Z_{i,s}$, and describes the relationship between the expected value, Z_i, and the uncertain parameter θ_i (Figure 1)

$$Z_i=r_i(\theta_i) \qquad i\in I'' \tag{11}$$

The basic point of parameter i, θ_i^{B}, is determined from this curve by the inverse function, r_i^{-1}

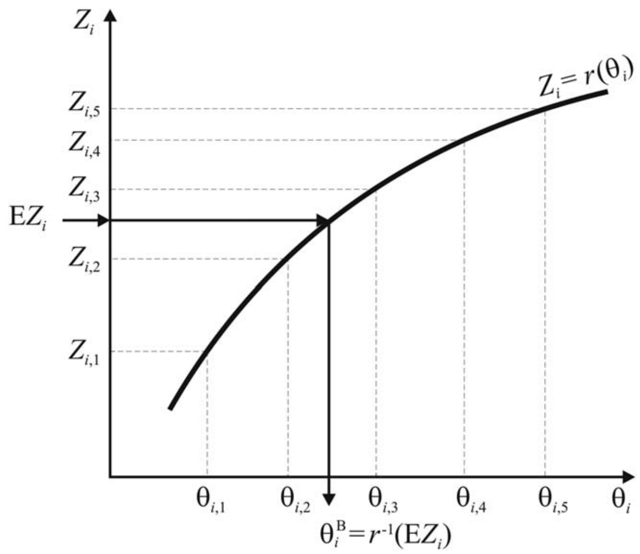

Figure 1. Schematic representation of the Central Basic Point determination.

$$\theta_i^{\rm B} = r_i^{-1}({\rm EZ}_i) \qquad i \in I'' \tag{12}$$

After determining the basic points for all the nonlinear uncertain parameters, a Central Basic Point is composed from its coordinates

$$\boldsymbol{\theta}^{\rm CBP} = \begin{cases} \theta_i^{\rm B} & \text{if} \quad i \in I'' \\ \theta_i^{\rm N} & \text{otherwise} \end{cases} \tag{13}$$

Testing the accuracy of an approximate stochastic result

If the problem is small enough to be solved by one of the rigorous methods, such as the Gaussian quadrature, the designs obtained by approximate methods could be tested by solving the DEP over the nonreduced set of scenarios at fixed values of the first-stage variables $\mathbf{d}^*$:

$$\begin{aligned} &{\rm EZ}(\mathbf{d}^*) = \max_{\mathbf{x}_s, \mathbf{z}_s} \left[-f^0(\mathbf{d}^*) + \sum_s \mathbf{p}_s f(\mathbf{x}_s, \mathbf{z}_s, \boldsymbol{\theta}_s) \right] \\ &\text{s.t.} \quad \left. \begin{array}{l} \mathbf{h}(\mathbf{d}^*, \mathbf{x}_s, \mathbf{z}_s, \boldsymbol{\theta}_s) = 0 \\ \mathbf{g}(\mathbf{d}^*, \mathbf{x}_s, \mathbf{z}_s, \boldsymbol{\theta}_s) \le 0 \\ \mathbf{d}^* \ge \mathbf{g}_{\rm d}(\mathbf{x}_s, \mathbf{z}_s, \boldsymbol{\theta}_s) \end{array} \right\} \quad s \in S \\ &\qquad \mathbf{x}_s, \mathbf{z}_s \in \Re^+ \end{aligned} \tag{NDEP}$$

Table 1. Uncertain Parameters of HEN Example

i	θ	Unit	$\theta_i^{\rm LO}$	$\theta_i^{\rm N}$	$\theta_i^{\rm UP}$	$N(\mu,\sigma)$
1	T_1	K	616	621	626	$N(621, 1.666)$
2	T_3	K	373	388	403	$N(388, 5)$
3	T_5	K	570	583	596	$N(583, 4.333)$
4	CF_1	kW/K	23	28	33	$N(28, 1.666)$
5	U_1	kW/(m^2 K)	0.5	0.7	0.9	$N(0.7, 0.066)$

where $\mathbf{d}^*$ represents those fixed design variables obtained by the approximate method, and S is a nonreduced set of scenarios, composed by, e.g., Gaussian quadrature points of all uncertain parameters plus the extreme points of uncertain parameters' intervals.

HEN Example

This problem involves the design of a flexible HEN with two hot streams, two cold streams, and five heat exchangers. The HEN is given in Figure 2, and the deterministic mathematical model in Appendix. Five uncertain parameters are defined in Table 1: the inlet temperatures of hot stream H1 (T_1), cold stream C1 (T_3), and hot stream H2 (T_5), the heat capacity flow rate of hot stream H1 (CF_1), and the overall heat-transfer coefficient of heat exchanger 1 (U_1). Each parameter was defined with the lower, nominal and upper values, and normal distribution with the mean μ and standard deviation σ, as shown in Table 1. Please note that number of uncertain parameters was limited to five for the sole reason that the problem could be solved by the rigorous Gaussian quadrature method for comparison. On solving this problem for one single scenario, i.e., the nominal values of uncertain parameters, the minimum total annual cost $Z = 597{,}235$ EUR/yr is obtained, and the inflexible design with flexibility index close to 0 (Table 2, first column).

It was assumed that the uncertain parameters are distributed independently, and that flexibility of HEN must be guaranteed within the uncertainty region defined by the lower and upper bounds. The areas of five heat exchangers ($A_1 \ldots A_5$) were declared as the first-stage design variables.

Testing the interactions

The interactions in the model were checked by applying a brute force Monte Carlo approach using 500 × 500 randomly selected combinations of uncertain parameters within the double loop, and computing the main and total effects of the uncertain parameters for five design variables (Table 3). It can be seen in Table 3 that the parameters T_3, CF_1, and

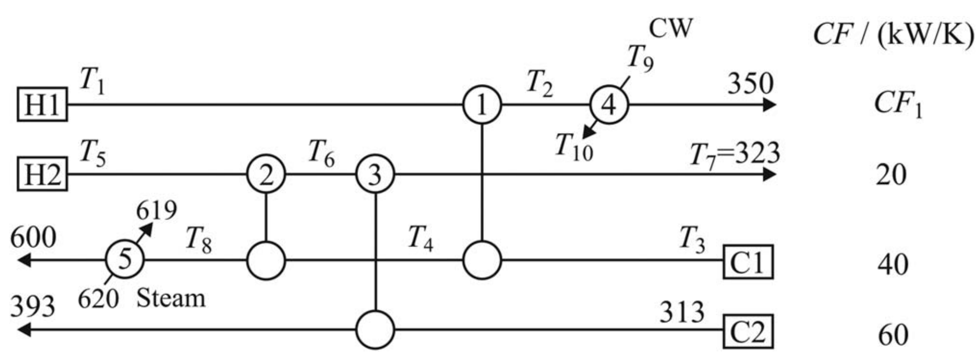

Figure 2. Heat exchanger network.

Table 2. Results of HEN Example

	Nominal point	Gaussian quadrature	CBP + critical points
EZ (EUR/yr)	597,235	639,329	634,608
EZ($\mathbf{d}^*$) (EUR/yr)	n.a.	639,329	663,852
Flexibility index	0.00000268	1.00000022	1.00000031
A_1 (m^2)	439.64	406.96	436.99
A_2 (m^2)	31.22	58.88	28.89
A_3 (m^2)	121.42	121.42	121.42
A_4 (m^2)	28.09	73.46	86.46
A_5 (m^2)	44.88	66.80	66.67
ΣA (m^2)	665.25	727.52	740.43
No. of scenarios	1	3157	5
No. of variables	15	28,421	51
No. of constraints	19	56,829	91
CPU (s)	0.047	139	0.062

U_1 have the largest total effects (underlined numbers). The parameter CF_1 was determined as the most interacting one because of the largest total effects, and the largest differences between the total and main effects. Anyway, the sums of the main effects amounted from 89.4% to 98.4%, which indicated the presence of weak nonlinear interactions, and implied that one-at-a-time sensitivity analysis could be used for scenario reduction.

Stochastic approach with Gaussian quadrature

The problem was then solved stochastically by Gaussian quadrature. Note that this was only possible because of low number of uncertain parameters. Quadrature points were defined for each uncertain parameter by the following formula

$$\theta_i^g = \theta_i^{\mathrm{LO}} + t^g\left(\theta_i^{\mathrm{UP}} - \theta_i^{\mathrm{LO}}\right) \quad (14)$$

where t^g are the parameters for the quadrature points calculation. In this case, five quadrature points were considered for each parameter with probabilities p^g corresponding to normal distribution as shown in Table 4. Combinations of five Gaussian quadrature points for five uncertain parameters yielded $5^5 = 3125$ scenarios whose probabilities were calculated as products of individual probabilities so that their total sum equals one. As Gaussian points do not include the lower and upper values of uncertain parameters, 32 extreme points (vertices) with zero probabilities were added in order to cover the specified uncertainty region, yielding altogether a set of $3125 + 32 = 3157$ scenarios.

On solving the DEP problem using 3157 scenarios, the design at a minimum expected cost of 639,329 EUR/yr was obtained. The detailed results are presented in Table 2 (column 2). Due to the nonconvex mathematical model, the flexibility of optimal design was tested over 10,000 randomly

Table 4. Parameters and Probabilities for Gaussian Quadrature Points

g	1	2	3	4	5
t^g	0.04691008	0.23076534	0.5	0.76923466	0.95308992
p^g	0.00700425	0.15449291	0.67700567	0.15449291	0.00700425

selected points using Monte Carlo sampling, which revealed that feasible operation could be achieved at the selected combinations of uncertain parameters. The flexibility index was determined close to 1.

Approximate stochastic solution with reduced set of scenarios

Sensitivity analysis was performed sequentially at the lower, nominal, and upper values of each uncertain parameter. By observing those values of uncertain parameters at which the design variables reached the largest values, the following matrix A was formulated:

		T_1	T_3	T_5	CF	U_1
	max A_1	3	1	1	3	1
	max A_2	2	2	2	2	2
$A =$	max A_3	0	0	0	0	0
	max A_4	2	2	2	2	1
	max A_5	1	1	1	1	1

a. Identification of critical points. Four critical points were then identified based on the matrix A

$$\boldsymbol{\theta}_1^{\mathrm{c}} = \left(T_1^{\mathrm{N}}, T_3^{\mathrm{LO}}, T_5^{\mathrm{LO}}, \mathrm{CF}^{\mathrm{N}}, U_1^{\mathrm{LO}}\right) = (621, 373, 570, 28, 0.5)$$

$$\boldsymbol{\theta}_2^{\mathrm{c}} = \left(T_1^{\mathrm{UP}}, T_3^{\mathrm{UP}}, T_5^{\mathrm{UP}}, \mathrm{CF}^{\mathrm{UP}}, U_1^{\mathrm{UP}}\right) = (626, 403, 596, 33, 0.9)$$

$$\boldsymbol{\theta}_3^{\mathrm{c}} = \left(T_1^{\mathrm{UP}}, T_3^{\mathrm{UP}}, T_5^{\mathrm{UP}}, \mathrm{CF}^{\mathrm{UP}}, U_1^{\mathrm{LO}}\right) = (626, 403, 596, 33, 0.5), \text{ and}$$

$$\boldsymbol{\theta}_4^{\mathrm{c}} = \left(T_1^{\mathrm{LO}}, T_3^{\mathrm{LO}}, T_5^{\mathrm{LO}}, \mathrm{CF}^{\mathrm{LO}}, U_1^{\mathrm{LO}}\right) = (616, 373, 570, 23, 0.5)$$

b. Determination of the Central Basic Point. It was established that the uncertain parameters T_1, T_3, and T_5 affected the objective function linearly. The coordinates of these parameters within the CBP were fixed at their nominal values. The effects of CF and U_1 on the objective variable were nonlinear, and the basic coordinates of CF and U_1 were determined according to the procedure described in the section Calculation of the Central Basic Point, yielding the following CBP

$$\boldsymbol{\theta}^{\mathrm{CBP}} = (T_1, T_3, T_5, \mathrm{CF}, U_1) = (621, 388, 583, 27.754371, 0.685464)$$

Table 3. Main and Total Effects (%) of Uncertain Parameters in HEN Example*

	A_1		A_2		A_3		A_4		A_5	
	Main	Total	Main	Total	Main	Total	Main	Total	Main	Total
T_1	0.6	3.0	1.3	2.4	0	0	0.1	0.4	1.4	1.5
T_3	2.4	4.5	1.7	2.6	0	0	45.2	<u>47.0</u>	2.3	2.4
T_5	0.0	0.0	3.9	4.6	0	0	0.0	<u>0.0</u>	5.4	5.5
CF_1	53.6	<u>58.7</u>	85.6	<u>88.2</u>	0	0	44.0	<u>45.3</u>	89.3	<u>89.7</u>
U_1	38.4	<u>38.7</u>	0.0	<u>0.1</u>	0	0	0.1	<u>0.1</u>	0.0	<u>0.0</u>
Sum	95.0	105.0	92.7	97.9	0	0	89.4	92.8	98.4	99.1

*Around 80 h of real time was needed to calculate the main and total effects for five uncertain parameters using an average PC.

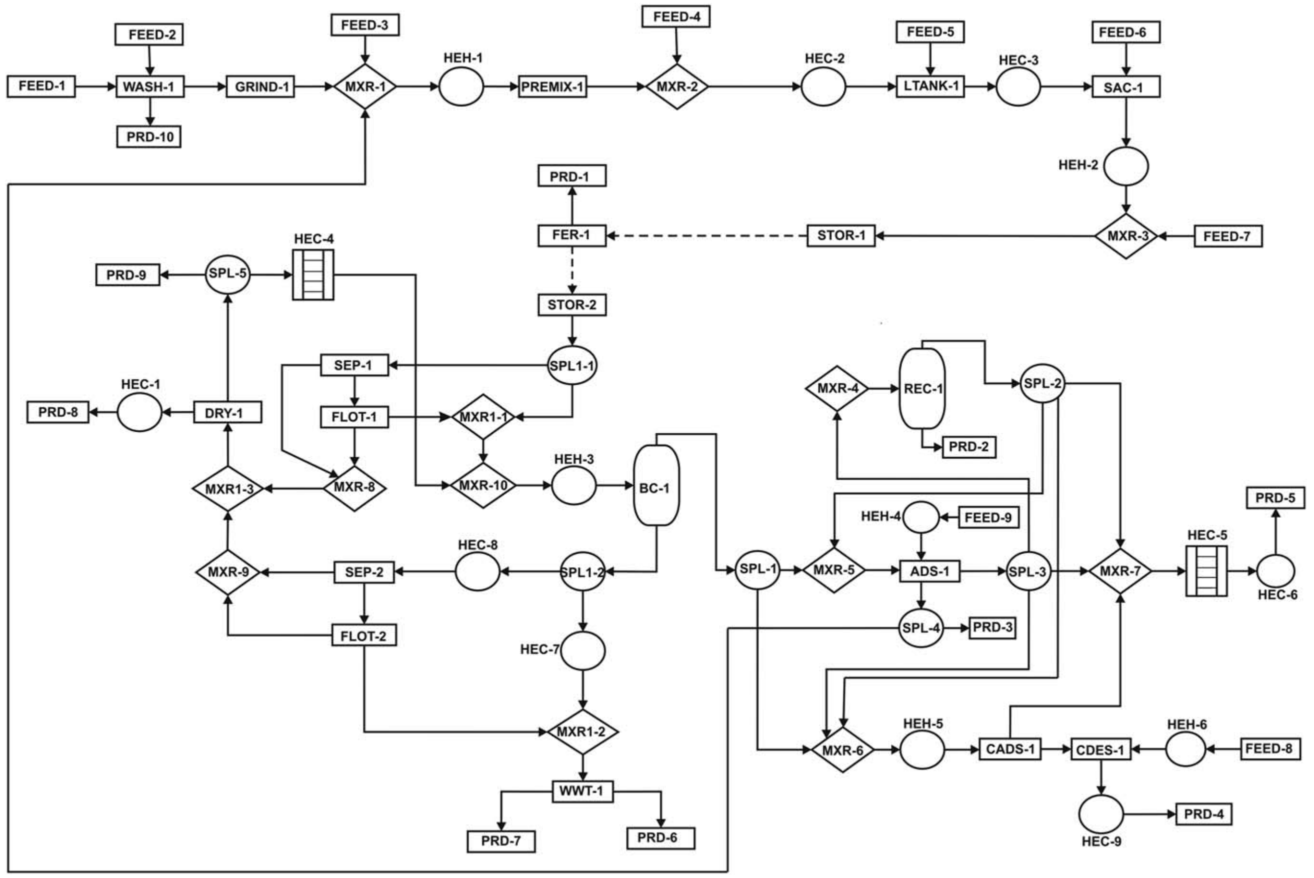

Figure 3. Superstructure of bioethanol process.

On solving the problem (RDEP) at the CBP and four critical points, the result 634,608 EUR/yr was obtained (last column in Table 2). The flexibility of this design was 1, however, the approximate objective cost (634,608) was somewhat underestimated in relation to the Gaussian result (639,329). As during optimization, the second-stage variables were only minimized at the CBP, an optimistic result was expected. Fixing the values of the heat exchanger areas and re-solving the nonreduced problem (NDEP) over 3157 scenarios (3125 Gaussian points and 32 vertices), the exact stochastic expected value of this design was obtained, which is 663,852 EUR/yr. This result overestimated the Gaussian result (639,329) by 3.8%. The size of the reduced model and the CPU time usage were smaller by several orders of magnitudes compared to the Gaussian quadrature model.

It should be mentioned, that an identical result would be obtained if all 32 vertices were used instead of four critical points only. This confirms that the correct critical points were selected by applying the sensitivity analysis approach to HEN problem with moderate interactions between uncertain parameters. Actually, only vertices $\boldsymbol{\theta}_3^c$ and $\boldsymbol{\theta}_4^c$ were critical, while the first two points were redundant.

Bioethanol Case Study

The case study process produces 180,000 t/yr of bioethanol. The process superstructure by Karuppiah et al.[21] is shown in Figure 3. Corn grains (FEED-1) are broken by means of physical treatment in the grinding unit (GRIND-1), and thermal treatment with steam (FEED-4). During the biological stages (liquefaction and saccharification, LTANK-1, and SAC-1), the starch is converted into sugars that are further fermented into ethanol (FER-1). The diluted ethanol solution is concentrated in a beer column (BC-1), followed by further recovery of ethanol in the rectification (REC-1), adsorption (ADS-1), and/or molecular sieves (CADS-1). The slurry product from fermentation is dried by the use of a mechanical press (SEP-1, SEP-2), flotation (FLOT-1, FLOT-2), and a dryer (DRY-1), and used as livestock feed. There are two discrete decisions in this case study: solid–liquid separation by mechanical press and flotation can take place before (SEP-1, FLOT-1) or after (SEP-2, FLOT-2) the beer column. In the latter case, the beer column provides the main separation of water from the stillage.

A deterministic MINLP model by Čuček and Kravanja[22] was used, while the objective function was transformed from profit maximization to the maximization of the net present value (NPV). The problem was solved by means of an advanced mixed-integer process synthesizer MipSyn.[23] The NPV of the optimal deterministic solution for a time period of 10 years was 367.9 M$, and the total capital cost 89.1 M$. Solid–liquid separation took place after the beer column. The deterministic model contained approximately 6400 variables and 6600 constraints.

In the next step, 71 uncertain parameters were defined for synthesis of the flexible bioethanol process under uncertainty: (a) 21 external parameters, e.g., product demand, prices of feedstocks, products and utilities, tax and discount rates, lifetime and depreciation periods, (b) 12 process parameters, e.g., feed compositions and feed temperatures, and (c) 38 model parameters, e.g., conversion coefficients, heat-transfer coefficients, tray efficiency, removal efficiencies, etc. Normal probability distributions were assumed for all parameters with the nominal values midway between the lower and upper bounds, and the lengths of the uncertain intervals equal to six standard deviations.

It was established by perturbing some uncertain parameters, that the deterministic design obtained at the nominal point was unable to tolerate deviations of some input data from the nominal values. A synthesis of the flexible bioethanol process flow sheet is presented in the continuation. As the superstructure involves only two discrete decisions (solid–liquid separation before or after the beer column), the optimal topology was selected first by using nominal values of uncertain parameters and critical points. The optimal structure obtained is then handled with the approximate stochastic design method.

Nominal synthesis of flexible bioethanol process

The critical points are determined first. In this case study, the total number of vertices was 2^{71}, which meant that DEP could not be solved by using all vertices. Rigorous integration or sampling methods were also impossible. The reduction of uncertain parameters and their scenarios was, therefore, needed.

The first reduction was achieved by assuming a perfect correlation between the removal efficiencies of various components in a wastewater treatment plant (WWT-1). Combining these parameters into a single one would reduce the total number from 71 to 56.

By performing sensitivity analysis, nonmonotonic influences on the design variables were determined for seven parameters: starch fraction in corn FEED-1, temperatures of FEED-1 and FEED-5 (enzyme), conversions in the reactor (FER-1), and moisture content in the flotation input stream. Monotonic influence was recognized for 16 parameters, e.g., demand of bioethanol product (PRD-5), overall heat-transfer coefficients in the heat exchangers (HEH and HEC), several conversions in the reactor, several input temperatures, and a pressure drop in the beer column (BC-1). The monotonic parameters were fixed at their critical bounds. The influences of others, especially external uncertain parameters such as the cost of electricity, feeds and products, depreciation period, discount rate, tax rate, etc., were negligible.

A matrix A (Eq. 5) was formed for seven parameters with nonmonotonic influences on design variables, yielding three critical vertices.

MINLP synthesis of the significantly reduced DEP problem was performed at the nominal point and three critical vertices. The same optimal topology was obtained as with the deterministic model, i.e., the mechanical press and flotation after the beer column. The NPV of the optimal flexible solution was 357.2 M$ which was lower than the NPV of the inflexible solution (367.9 M$). The total capital cost amounted to 97.3 M$ and was higher than in the case of the inflexible design (89.1 M$). This indicated that the inflexible solutions were not only unable to tolerate the deviations of uncertain parameters but also underestimated the investment cost and overestimated the NPV. The size of the problem increased to around 25,700 variables and 25,700 constraints.

Approximate stochastic design of flexible bioethanol process

It was determined during the sensitivity analysis of the optimal process topology that 28 parameters affected the objective function nonlinearly. The basic points were determined for these parameters, and combined into the CBP. This step was computationally the most demanding as the decomposed optimization problems for each of the uncertain parameters contained around 50,000 continuous variables and about the same number of constraints.

Finally, the deterministic-equivalent problem was solved at the basic point and the critical vertices. The NPV of the approximate stochastic solution amounted to 356.8 M$, and was slightly lower than that obtained at the nominal point in the previous step (357.2 M$). This indicated that a somewhat optimistic result was obtained at the nominal point. The investment cost was very close to the nominal solution, i.e., 97.2 M$ vs. 97.3 M$. Although these results cannot be verified by other methods, it could be expected that the design obtained is flexible for specified deviations of uncertain parameters, assuming the correct set of critical points were selected.

Conclusions

This article presented an approach for the design and synthesis of process flow sheets with a large number of uncertain parameters. The main feature of this approach is the significant reduction of scenarios for deterministic equivalent problem. The reduction is achieved by means of prescreening sensitivity analyses that evaluate the influence of uncertain parameters on the first-stage design variables and the objective function. Parameters with nonspecific influences on the design variables are used for the determination of critical points. Those parameters with distinctive nonlinear influences on the objective function are used to determine the central basic point in order to take into account any deviation from the nominal point. The deterministic equivalent problem is finally solved at the critical points for feasibility, while the expected objective function is approximated at one single scenario, i.e., the central basic point.

The proposed approach is suitable for problems with none or weak interactions between uncertain parameters, while the flexibility of final process designs could be strictly guaranteed only for convex problems. Smaller case studies that can be solved by means of more rigorous methods confirmed that the proposed approximate approach provides flexible designs with fair approximation of the stochastic result at tremendously-reduced computational effort. Therefore, we believe this is one of the rarer ways of handling design problems regarding large-scale process flow sheets with many uncertain parameters. The critical step, however, remains the identification of critical points, especially in those problems with significantly interacting uncertain parameters. The experiences showed that this method based on sensitivity analysis often generates some redundant critical points. The future work will be oriented toward improving the preciseness of critical points identification.

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Appendix

$$z=\min\left[1846\left(A_1^{0.65}+A_2^{0.65}+A_3^{0.65}+A_4^{0.65}\right)+2350A_5^{0.65}\right]+20\Phi_c+230\Phi_h$$

s.t.

$$CF_1(T_1-T_2)=40(T_4-T_3)$$
$$20(T_5-T_6)=40(T_8-T_4)$$
$$20(T_6-T_7)=60(393-313)$$

$$\Phi_c=CF_1(T_2-350)$$
$$\Phi_h=40(600-T_8)$$
$$T_{10}=T_9+\Phi_c/40$$

$$A_1=\frac{40(T_4-T_3)}{U_1\Delta_{\ln}T_1}\quad \Delta_{\ln}T_1=\frac{(T_1-T_4)-(T_2-T_3)}{\ln\frac{T_1-T_4}{T_2-T_3}}$$

$$A_2=\frac{20(T_5-T_6)}{0.7\cdot\Delta_{\ln}T_2}\quad \Delta_{\ln}T_2=\frac{(T_5-T_8)-(T_6-T_4)}{\ln\frac{T_5-T_8}{T_6-T_4}}$$

$$A_3=\frac{20(T_6-T_7)}{0.7\cdot\Delta_{\ln}T_3}\quad \Delta_{\ln}T_3=\frac{(T_6-393)-(T_7-313)}{\ln\frac{T_6-393}{T_7-313}}$$

$$A_4=\frac{\Phi_c}{0.7\cdot\Delta_{\ln}T_4}\quad \Delta_{\ln}T_4=\frac{(T_2-T_{10})-(350-T_9)}{\ln\frac{T_2-T_{10}}{350-T_9}}$$

$$A_5=\frac{\Phi_h}{1\cdot\Delta_{\ln}T_5}\quad \Delta_{\ln}T_5=\frac{(620-600)-(619-T_8)}{\ln\frac{620-600}{619-T_8}}$$

$$T_2\geq T_3+1\quad T_6\geq 393+1$$
$$T_6\geq T_4+1\quad T_7=323$$
$$T_5\geq T_8+1\quad T_8\leq 619-1$$

$$T_9=298\ \text{K},\ T_1=621\ \text{K},\ T_3=388\ \text{K},\ T_5=583\ \text{K}$$
$$CF_1=28\ \text{kW/K},\quad U_1=0.7\ \text{kW/(m}^2\cdot\text{K)}$$

$$T_2,T_4,T_6,T_7,T_8,T_{10},\Phi_c,\Phi_h,A_1,\ldots,A_5,\Delta_{\ln}T_1,\ldots,\Delta_{\ln}T_5\geq 0$$

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